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A NEW HORIZON FOR MULTIPLE ATTRIBUTE GROUP DECISION MAKING PROBLEMS WITH VAGUE SETS

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ABSTRACT

Since its entry in the literature, vague set theory has received more and more attention, because many of the real life problems are information in the form of vague values. For Multiple Attribute Group Decision Making (MAGDM) problems where the attribute weights and the expert weights are real numbers and the attribute values take the form of vague values, a new approach is introduced in this paper. The IVOWA operator is introduced and utilized for aggregating the vague information. The induced vague ordered weighted averaging operator (IVOWA) for vague sets is introduced and a MAGDM model is developed based on the IVOWA operator and the vague weighted averaging (VWA) operator. A simple illustration is presented to show the effectiveness of the proposed mode and a comparison of the proposed model is made with an existing method.

KEYWORDS: Multiple Attribute Group Decision Making, Induced Vague Ordered Weighted Averaging Operator (IVOWA), Vague Sets

INTRODUCTION

Since the theory of fuzzy sets [20] was proposed in 1965, it has been applied in many uncertain information processing problems successfully, since in the real world there is vague information about different applications. In [8], Gau & Buehrer pointed out the drawback of using the single membership value in fuzzy set theory. In order to tackle this problem, they [8] proposed the notion of Vague Sets (VSs), which allow using interval-based membership instead of using point-based membership as in FSs. The interval-based membership generalization in VSs is more expressive in capturing vagueness of data. However, VSs are shown to be equivalent to that of IFSs [6]. For this reason, the interesting features for handling vague data that are unique to VSs are largely ignored. Atanasov [4, 5] proposed Intuition fuzzy set theory. Gau and Buehrer [8] proposed the concept of Vague set. Bustince & Burillo [6] proposed that Vague set was intuitionistic fuzzy sets and unified the intuition fuzzy set and the Vague set. As the Vague set [6] took the membership degree, non-membership degree and hesitancy degree into account, and has more ability to deal with uncertain information than traditional fuzzy set, lots of scholars pay attentions to the research of Vague set. Atanassov and Gargov [5] extended the intuition vague set and proposed the concept of interval intuition vague set, also named interval Vague set.

A Vague Set (VS), as well as an Intuitionistic Fuzzy Set (IFS), is a further generalization of an fuzzy set. Instead of using point-based membership as in FSs, interval-based membership is used in a VS. The interval-based membership in VSs is more expressive in capturing vagueness of data. In the literature, the notions of IFSs and VSs are regarded as

equivalent, in the sense that an IFS is isomorphic to a VS. Furthermore, due to such equivalence and IFSs being earlier known as a tradition, the interesting features for handling vague data that are unique to VSs are largely ignored. Decision-making is the process of finding the best option from all of the feasible alternatives. Sometimes, decision-making problems considering several criteria are called multi-criteria decision-making (MCDM) problems. The MCDM problems may be divided into two kinds. One is the classical MCDM problems [10, 11], among which the ratings and the weights of criteria are measured in crisp numbers. Another is the fuzzy multiple criteria decision-making (FMCDM) problems, among which the ratings and the weights of criteria evaluated on imprecision and vagueness are usually expressed by linguistic terms, fuzzy numbers or intuition fuzzy numbers.

A MAGDM problem is to find a desirable solution from a finite number of feasible alternatives assessed on multiple attributes, both quantitative and qualitative. In order to choose a desirable solution, the decision maker often provides his/her preference information which takes the form of numerical values, such as exact values, interval number values and fuzzy numbers. However, under many conditions, numerical values are inadequate or insufficient to model real-life decision problems. Indeed, human judgments including preference information may be stated in vague information. Hence, MAGDM problems under vague environment [10, 12, 13, 14, 15, 16, 17] is an interesting area of study for researchers in the recent days.

Different types of aggregation operators are found in the literature for aggregating the information. A very common aggregation method is the ordered weighted averaging (OWA) operator. It provides a parameterized family of aggregation operators that includes as special cases the maximum, the minimum and the average criteria. Since its appearance, the OWA operator has been used in a wide range of applications. Induced intuitionistic fuzzy operators are already in the literature [18, 19]. In this paper we propose the weighted averaging operator, ordered weighted averaging operator and the induced ordered weighted averaging operator for vague sets. We also propose a general model for decision making utilising these operators for vague sets together with a new distance function defined based on the distance functions from [9].

PRELIMINARIES OF VAGUE SETS

In this section, we some basic concepts related to VSs are discussed. Let U be a classical set of objects, called the universe of discourse, where an element of U is denoted by u.

Definition (Vague Set)

A vague set A in a universe of discourse U is characterized by a true membership function, t_A, and a false membership function, f_A , as follows [7]:

$t_A : U \rightarrow [0, 1]$, $f_A : U \rightarrow [0, 1]$, and $t_A(u) + f_A(u) \leq 1$, where $t_A(u)$ is a lower bound on the grade of membership of u derived from the evidence for u, and $f_A(u)$ is a lower bound on the grade of membership of the negation of u derived from the evidence against u. Suppose $U = \{u_1, u_2, ..., u_n\}$. A vague set A of the universe of discourse U can be represented by

$$A=\sum_{i=1}^{n}[t(u_i),1-f(u_i)]/u_i, 0 \leq t(u_i) \leq 1-f(u_i) \leq 1, 1 \leq i \leq n .$$

In other words, the grade of membership of u_i is bounded to a subinterval $[t_A(u_i), 1 - f_A(u_i)]$ of [0, 1]. Thus, VSs are a generalization of FSs, since the grade of membership $\mu_A(u)$ of u in the above definition may be inexact in a VS.

Basic Definitions and Operations in Vague Sets

Let x, y be the two vague values in the universe of discourse U, $x = [t_x, 1 - f_x]$, $y = [t_y, 1 - f_y]$, where $t_x, f_x, t_y, f_y \in [0,1]$ and $t_x + f_x \le 1, t_y + f_y \le 1$; the operation and relationship between vague values is defined as follows [1, 2, 3]:

Definition

The minimum operation of vague values x and y is defined by

$$x \wedge y = [\min(t_x, t_y), \min(1 - f_x, 1 - f_y)]$$

$$= [\min(t_x, t_y), 1 - \max(1 - f_x, 1 - f_y)]$$

Definition

The maximum operation of vague values x and y is defined by

$$x \vee y = [\max(t_x, t_y), \max(1 - f_x, 1 - f_y)]$$

$$= [\max(t_x, t_y), 1 - \min(1 - f_x, 1 - f_y)]$$

Definition

The complement of vague value x is defined by

$$\overline{x} = [f_x, 1 - t_x]$$

Let A, B be two VSs in the universe of discourse $U = \{u_1, u_2, \ldots, u_n\}$,

$A = \sum_{i=1}^{n} [t_A(u_i), 1 - f_A(u_i)]/u_i$ and $B = \sum_{i=1}^{n} [t_B(u_i), 1 - f_B(u_i)]/u_i$, then the operations between VSs are defined as follows.

The intersection of VSs A and B is defined by

$$A \cap B = \sum_{i=1}^{n} \{[t_A(u_i), 1 - f_A(u_i)] \wedge [t_B(u_i), 1 - f_B(u_i)]\}/u_i$$

The union of vague sets A and B is defined by

$$A \cup B = \sum_{i=1}^{n} \{[t_A(u_i), 1 - f_A(u_i)] \vee [t_B(u_i), 1 - f_B(u_i)]\}/u_i$$

The complement of vague set A is defined by

$$\overline{A} = \sum_{i=1}^{n} [f_A(u_i), 1 - t_A(u_i)]/u_i.$$

Definition

For vague value x = [t_x, 1 - f_x], define the defuzzification function to get the precise value as follows:

Dfzz (x) = t_x / (t_x + f_x).

Graphical Representation of a VS

As we can see that the difference between VSs and IFSs is due to the definition of membership intervals. We have [t_A(u), 1 – f_A(u)] for u in A (a VS), but < u_A(x), v_A(x) > for x in A (an IFS). Here the semantics of u_A is the same as with t_A and v_A is the same as with f_A. However, the boundary (1 − f_A) is able to indicate the possible existence of a data value, as already mentioned by An Lu and Wilfred [1,2,3]. This subtle difference gives rise to a simpler but meaningful graphical view of data sets. Figure 1 represents the existence of the vague data value in between the boundaries of t_A and (1- f_A).

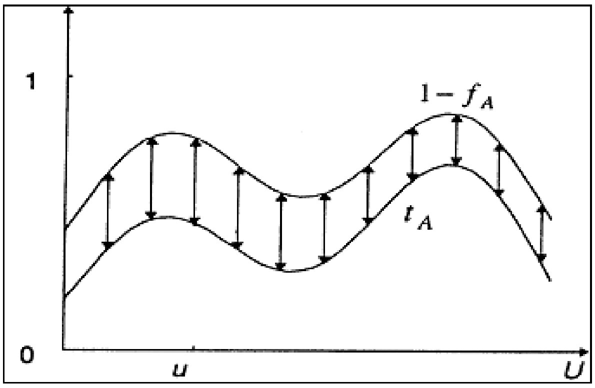

Figure 1: Geometrical Interpretation of a Vague Set

It can be seen that, the shaded part formed by the boundary in a given VS in Figure 1 naturally represents the possible existence of data. Thus, this “hesitation region” corresponds to the intuition of representing vague data. The choice of the membership boundary also has interesting implications on modelling relationship between vague data. In the following, a new distance function is defined for ranking the alternatives in MAGDM problems.

Definition

Let $A=\left(t_A(x_i), f_A(x_i)\right)$, $B=\left(t_B(x_i), f_B(x_i)\right)$ be two vague values. Then the Euclidean distance between A and B is given as follows:

$$d(A,B)=\sqrt{\frac{1}{2}\sum_{i=1}^{n}\left[(t_A(x_i)-t_B(x_i))^2+((1-f_A(x_i))-(1-f_B(x_i)))^2\right]}$$

THE IVOWA OPERATOR FOR GROUP DECISION MAKING

In this paper we define a new operator called the Induced Vague Ordered Weighted Averaging (IVOWA) operator for vague sets.

Definition

Let $a_j=(t_j, f_j), j=1,2,...,n$ be a collection of vague values, and let the vague weighted averaging operator

VWA is defined as: $VWA: Q^n \to Q$ if

$$VWA_{\omega}(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} \omega_j a_j = \left(1 - \prod_{j=1}^{n} (1-t_j)^{\omega_j}, \prod_{j=1}^{n} (1-f_j)^{\omega_j} \right)$$

Where $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ be the weight vector of $a_j = (t_j, f_j)$ and $\omega_j > 0$, $\sum_{j=1}^{n} \omega_j = 1$.

Definition

Let $a_j = (t_j, f_j), j = 1, 2, ..., n$ be a collection of vague values, then the Vague Ordered Weighted Averaging (VOWA) operator of dimension n is given by the mapping $VOWA: Q^n \to Q$, with an associated weight vector $\mathrm{w} = (\mathrm{w}_1, \mathrm{w}_2, ..., \mathrm{w}_n)^T$ such that

$$\mathrm{w}_j > 0 \quad \text{and} \quad \sum_{j=1}^{n} \mathrm{w}_j = 1.$$

$$VOWA_{\mathrm{w}}(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} \mathrm{w}_j a_{\sigma(j)} = \left(1 - \prod_{j=1}^{n} (1-t_{\sigma(j)})^{\mathrm{w}_j}, \prod_{j=1}^{n} (1-f_{\sigma(j)})^{\mathrm{w}_j} \right)$$

Where $(\pi(1), \pi(2), ..., \pi(n))$ is a permutation of (1,2,…,n) such that $\alpha_{\pi(j-1)} \geq \alpha_{\pi(j)}$ for all j=2,3,…,n.

Definition

The IVOWA operator is defined as follows:

$$IVOWA_{\mathrm{w}}\left(\langle u, a_1 \rangle, \langle u, a_2 \rangle, ..., \langle u, a_n \rangle \right) = \sum_{j=1}^{n} \mathrm{w}_j g_j = \left(1 - \prod_{j=1}^{n} (1-\bar{t}_j)^{\mathrm{w}_j}, \prod_{j=1}^{n} (1-\bar{f}_j)^{\mathrm{w}_j} \right)$$

Where $\mathrm{w} = (\mathrm{w}_1, \mathrm{w}_2, ..., \mathrm{w}_n)^T$ is a weighting vector such that $\mathrm{w}_j \in [0,1]$ and $\sum_{j=1}^{n} \mathrm{w}_j = 1$, $g_j = (\bar{t}_j, \bar{f}_j)$ is the a_i value of VOWA pair $\langle u_i, a_i \rangle$ having the j^th^ largest $u_i \in [0,1]$ and u_i in $\langle u_i, a_i \rangle$ is called as the order inducing variable and a_i is the vague value. The IVOWA operator satisfies the following properties:

Commutativity

$$IVOWA_{\mathrm{w}}\left(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, ..., \langle u_n, a_n \rangle \right) = IVOWA_{\mathrm{w}}\left(\langle u_1, a'_1 \rangle, \langle u_2, a'_2 \rangle, ..., \langle u_n, a'_n \rangle \right)$$

Where $\left(\langle u_1, a'_1 \rangle, \langle u_2, a'_2 \rangle, ..., \langle u_n, a'_n \rangle \right)$ is any permutation of $\left(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, ..., \langle u_n, a_n \rangle \right)$.

Idempotency

If $a_j = a$, where $a_j = (t_j, f_j), a = (t, f)$ for all j,

Then $IVOWA_w\left(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, ..., \langle u_n, a_n\rangle\right) = a$

Monotonicity

If $a_j \le a'_j$ for all j, then

$$IVOWA_w\left(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, ..., \langle u_n, a_n\rangle\right) \le IVOWA_w\left(\langle u_1, a'_1\rangle, \langle u_2, a'_2\rangle, ..., \langle u_n, a'_n\rangle\right)$$

PROPOSED MODEL OF MAGDM

Let $A = \{A_1, A_2, ..., A_n\}$ be a set of alternatives, $G = \{G_1, G_2, ..., G_n\}$ be the set of alternatives, $\omega = (\omega_1, \omega_2, ..., \omega_n)$ is the weighting vector of the attribute G_j, j=1,2,…,n, where $\omega_j \in [0,1]$, $\sum_{j=1}^{n} \omega_j = 1$. Let $D = \{D_1, D_2, ..., D_t\}$ be the set of decision makers, $V = (V_1, V_2, ..., V_n)$ be the weighting vector of the decision makers, with $V_k \in [0,1]$, $\sum_{k=1}^{t} V_k = 1$. Let $R_k = \left(\tilde{r}_{ij}^{(k)}\right)_{m\times n} = \left(t_{ij}^{(k)}, f_{ij}^{(k)}\right)_{m\times n}$ be the vague decision matrix, where $t_{ij}^{(k)}$ is the degree of the truth membership value that the alternative A_i satisfies the attribute G_j given by the decision maker D_k and $f_{ij}^{(k)}$ is the degree of false membership value that the alternative for the alternative A_i, where $t_{ij}^{(k)}, f_{ij}^{(k)} \subset [0,1]$ and, $t_{ij}^{(k)} + f_{ij}^{(k)} \le 1$, i=1,2,…,m, j=1,2,…,n, k=1,2,…,t

The developed model of MAGDM is given as follows:

Step 1: Utilize the vague decision matrix $R_k = \left(\tilde{r}_{ij}^{(k)}\right)_{m\times n} = \left(t_{ij}^{(k)}, f_{ij}^{(k)}\right)_{m\times n}$ and the IVOWA operator which has the associated weighting vector $\mathrm{w} = (\mathrm{w}_1, \mathrm{w}_2, ..., \mathrm{w}_n)^{\mathrm{T}}$

$\tilde{r}_{ij} = \left(t_{ij}, f_{ij}\right) = IVOWA_w\left(\left\langle V_1, \tilde{r}_{ij}^{(1)}\right\rangle, \left\langle V_2, \tilde{r}_{ij}^{(1)}\right\rangle, ..., \left\langle V_t, \tilde{r}_{ij}^{(1)}\right\rangle\right)$, i=1,2,…,m, j=1,2,…,n, to aggregate into a collective decision matrix $R_k = \left(\tilde{r}_{ij}^{(k)}\right)_{m\times n}$, where $\mathrm{V} = (\mathrm{V}_1, \mathrm{V}_2, ..., \mathrm{V}_t)$ be the weighting vector of the decision maker.

Step 2: Utilizing the information from the collective decision matrix $R_k = \left(\tilde{r}_{ij}^{(k)}\right)_{m\times n}$ and the VWA operator $\tilde{r}_i = \left(t_i, f_i\right) = VWA_\omega\left(\tilde{r}_{i1}, \tilde{r}_{i2}, ..., \tilde{r}_{in}\right)$, i=1,2,…,m, derive the collective overall preference values of the alternative A_i, when $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is the weighting vector of the attributes.

Step 3: Calculate the distance between the collective overall preference values $\tilde{r}_i$ and the positive ideal vague value $\tilde{r}^+$, or the negative ideal vague value $\tilde{r}^-$, where $\tilde{r}^+ = (1,0)$ and $\tilde{r}^- = (0,1)$. Using the Euclidean distance function we can find the distances between the collective overall preference values $\tilde{r}_i$ and the positive ideal vague value $\tilde{r}^+$ as follows:

$$d(\tilde{r}_i,\tilde{r}^+) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left[(t_{\tilde{r}_i}(x_i) - t_{\tilde{r}^+}(x_i))^2 + ((1-f_{\tilde{r}_i}(x_i)) - (1-f_{\tilde{r}^+}(x_i)))^2\right]}$$

Step 4: Rank all the alternatives A_i, where i = 1, 2,…,m and select the best one in accordance with the distance obtained in step 3.

NUMERICAL ILLUSTRATION

Suppose an investment company, wanting to invest a sum of money in the best option, and there is a panel with five possible alternatives to invest the money;

A_1 is an IT company,

A_2 is a multinational company,

A_3 is a tools company,

A_4 is an airlines company and

A_5 is an automobile company.

The investment company must take a decision according to the four following attributes; G_1 is the risk analysis, G_2 is the growth analysis, G_3 is the socio-political impact analysis and G_4 is the environmental impact analysis. The five possible alternatives A_i, where i = 1,2,…,m, are to be evaluated by three decision makers whose weighting vector is $V = (0.35,0.40,0.25)^T$ under the above said four attributes whose weighting vector is $\omega = (0.2,0.1,0.3,0.4)^T$, which gives the decision matrices of vague values $R_k = \left(\tilde{r}_{ij}^{(k)}\right)_{5\times 4}, k = 1,2,3$:

$$R_1 = \begin{bmatrix} (0.4873, 0.7256) & (0.5221, 0.7222) & (0.6286, 0.8312) & (0.4427, 0.9986) \\ (0.3271, 0.9001) & (0.6676, 0.5413) & (0.4261, 0.8126) & (0.7710, 0.9442) \\ (0.5238, 0.8011) & (0.4278, 0.5261) & (0.5527, 0.6216) & (0.5687, 0.7981) \\ (0.7218, 0.6283) & (0.7213, 0.8912) & (0.8311, 0.9219) & (0.6626, 0.8215) \\ (0.6257, 0.7983) & (0.8321, 0.9426) & (0.6256, 0.7119) & (0.4136, 0.6295) \end{bmatrix}$$

$$R_2 = \begin{bmatrix} (0.4351, 0.7846) & (0.5121, 0.7221) & (0.1009, 0.6221) & (0.2217, 0.7184) \\ (0.6321, 0.8221) & (0.6226, 0.8108) & (0.3009, 0.5129) & (0.6225, 0.9105) \\ (0.5387, 0.9105) & (0.4124, 0.7216) & (0.5010, 0.7101) & (0.4491, 0.5426) \\ (0.7317, 0.8119) & (0.5221, 0.8001) & (0.2091, 0.4104) & (0.2101, 0.4110) \\ (0.5273, 0.6217) & (0.3125, 0.7278) & (0.4728, 0.7182) & (0.6210, 0.8109) \end{bmatrix}$$

$$R_3 = \begin{bmatrix} (0.3198, 0.8279) & (0.4419, 0.9816) & (0.2211, 0.5221) & (0.6661, 0.7027) \\ (0.7726, 0.8901) & (0.6245, 0.7815) & (0.6216, 0.8225) & (0.7101, 0.9005) \\ (0.5201, 0.7287) & (0.5821, 0.6286) & (0.7117, 0.9211) & (0.6105, 0.9117) \\ (0.3247, 0.4821) & (0.7139, 0.8148) & (0.4212, 0.5334) & (0.5529, 0.7217) \\ (0.7351, 0.9113) & (0.8001, 0.9112) & (0.2221, 0.6121) & (0.4214, 0.5005) \end{bmatrix}$$

Step 1: Utilizing the decision information given in the matrix $R_k = \left(\tilde{r}_{ij}^{(k)}\right)_{5\times 4}, k = 1, 2, 3$ and the *IVOWA* operator which has the associated weighting vector w = (0.2, 0.35, 0.45) ^{T}, we get a collective decision matrix $R_k = \left(\tilde{r}_{ij}^{(k)}\right)_{5\times 4}$

$$R = \begin{bmatrix} (0.4229, 0.7718) & (0.4933, 0.8041) & (0.4256, 0.6666) & (0.5020, 0.8267) \\ (0.5921, 0.8805) & (0.6442, 0.6673) & (0.4540, 0.7443) & (0.7252, 0.9219) \\ (0.5255, 0.7951) & (0.4847, 0.5964) & (0.6080, 0.7326) & (0.5629, 0.7741) \\ (0.6233, 0.6028) & (0.6868, 0.8452) & (0.6461, 0.6475) & (0.5586, 0.6835) \\ (0.6525, 0.7954) & (0.7634, 0.8845) & (0.4821, 0.6764) & (0.4651, 0.6111) \end{bmatrix}$$

Step 2: Utilizing the *VWA* operator, we obtain the collective overall preference values $\tilde{r}_i$ of the alternatives A_i, where i = 1, 2,…,5.

$$\tilde{r}_1 = (0.4637, 0.7622),\ \tilde{r}_2 = (0.6313, 0.8294),\ \tilde{r}_3 = (0.5628, 0.7458)$$
$$\tilde{r}_4 = (0.6133, 0.6699),\ \tilde{r}_5 = (0.5521, 0.6891).$$

Step 3: Calculating the distances between the collective overall preference values $\tilde{r}_i$ and the positive ideal vague value $\tilde{r}^+ = (1, 0)$. The distances calculated from the following distance function

$$d(\tilde{r}_i, \tilde{r}^+) = \sqrt{\frac{1}{2}\sum_{i=1}^{n} \left[(t_{\tilde{r}_i}(x_i) - t_{\tilde{r}^+}(x_i))^2 + ((1 - f_{\tilde{r}_i}(x_i)) - (1 - f_{\tilde{r}^+}(x_i)))^2 \right]}$$

are given by:

$$d_1(\tilde{r}_1, \tilde{r}^+) = 0.6142,\ d_2(\tilde{r}_2, \tilde{r}^+) = 0.5825,\ d_3(\tilde{r}_3, \tilde{r}^+) = 0.5285,\ d_4(\tilde{r}_4, \tilde{r}^+) = 0.4231,\ d_5(\tilde{r}_5, \tilde{r}^+) = 0.4776.$$

Step 4: Rank the alternatives based on the shortest distance:

$A_4 < A_5 < A_3 < A_2 < A_1$

Hence A_4 is the best alternative.

Let us consider the replacing of Step 3 with the correlation coefficient proposed in [14] and [17]. Then the ranking order of the alternatives is obtained as follows:

$A_2 > A_4 > A_3 > A_5 > A_1$

Hence A_2 is the best alternative.

From the comparison, it can be observed that there is a change in the ranking of the best alternatives. In the proposed method with a distance function, A_4 is the best alternative, and with the replacement of step 3 in the algorithm with methods as in [14] and [17] it can be seen that A_2 is the best alternative

CONCLUSIONS

In this paper we have presented the ordered weighted averaging operators in the context of vague set theory. Vague sets can better handle vagueness and uncertainty than intuitionistic fuzzy sets. Initially the vague weighted averaging (VWA) operator was developed and based on VWA operator, the vague ordered weighted averaging (VOWA) operator was introduced. Finally the induced ordered weighted averaging (I-VOWA) operator was developed and a MAGDM model was proposed using all the above proposed operators. The proposed model of MAGDM can be utilised based on any distance function present in the literature.

REFERENCES

1. An Lu, & Wilfred Ng, (2004) Managing merged data by vague functional dependencies, *Lecture Notes in Computer Science – 3288, Springer,* 259-272
2. An Lu, & Wilfred Ng, (2005) Vague sets or intuitionistic fuzzy sets for handling vague data: Which one is better, *Lecture Notes in Computer Science – 3716, Springer,* 401-416
3. An Lu, & Wilfred Ng, (2009) Maintaining consistency of vague databases using data dependencies, *Data and Knowledge Engineering, 68(7),* 622-641.Atanassov, K, (1986). Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20, 87- 96.
4. Atanassov, K, (1989). More on intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 33, 37- 46.
5. Atanassov, K, (1989). More on intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 33, 37- 46.
6. Bustince, H & Burillo, P., (1996). Vague sets are intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 79, 403-405.
7. Chen, S.M, & Tan JM, (1994). Handling multi-criteria fuzzy decision making problems based on vague set theory, *Fuzzy sets and systems*, 67, 163-172.
8. Gau, W.L & Buehrer, D.J., (1994). Vague sets, *IEEE Transactions on Systems, Man and Cybernetics, Part A*, 23, 610 - 614.

9. Grzegorzewski, P., (2004). Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on Hausdorff metric, *fuzzy sets and systems*, 148, 319-328.
10. Hong, D.H & Choi, C.H., (2000). multi-criteria fuzzy decision making problems based on vague set theory, *Fuzzy sets and systems*,114, 103-113.
11. Hwang, C.L & Yoon, K, (1981), Multiple Attributes Decision Making Methods and Applications, *Springer*, Berlin-Heidelberg.
12. Li, D.F., (2005). Multiple attribute decision making models and methods using intuitionistic fuzzy sets, *Journal of Comp Syst Sci*, 70, 73-85.
13. Power, D.J., (2013). Engineering effective decision support technologies: New models and applications. Hershey: IGI Global.
14. Robinson, J.P., Amirtharaj, E.C.H, (2011). A short primer on the Correlation coefficient of Vague sets, *International Journal of Fuzzy System Applications*, 1(2), 55-69
15. Robinson, J.P., Amirtharaj, E.C.H, (2012). Vague Correlation coefficient of Interval Vague sets, *International Journal of Fuzzy System Applications*, 2(1), 18-34.
16. Robinson, J.P, & Amirtharaj, E.C.H., (2013). A Strategic TOPSIS algorithm with correlation coefficient of interval vague sets. *International Journal of Computing Algorithm.* 2, 314-331.
17. Solairaju, A, Robinson, J.P., & Lenin, T., (2014). Applications of Transforming Vague Sets into Fuzzy Sets for Knowledge Management. *International Journal of Computing Algorithm.* 2(2), (in press)
18. Wei, G., (2010). Some arithmetic aggregation operators with intuitionistic trapezoidal fuzzy numbers and their application to group decision making, *Journal of Computers*, 5 (3), 345-351.
19. Wei, G., (2008). Induced intuitionistic fuzzy ordered weighted averaging operator and its application to MAGDM, *LNAI-5009*, *Springer-Verlag Berlin Heidelberg*, 2008
20. Zadeh, L.A, (1965). Fuzzy sets, *Inf Control*, 8, 338-356.